and without softening strips) are quite high (150–250 ksi) for suitable laminate patterns, so integral local build ups within a laminate are capable of providing adequate strength without additional reinforcement in most cases.

Knowing how increasing the ultimate strain capacity in the vicinity of a bolt hole has dramatically increased the joint strengths, it is not clear how going in the opposite direction can improve matters further. The problem is that the boron film has an ultimate tensile strain of only about half that of the 0° filaments, so one would expect the boron film to break long before the filaments could be loaded up, so aggravating the stress concentration problem. Indeed, the author has recorded a reduction in relative displacement δ prior to failure in his tests. The reason why an improvement in joint strength was demonstrated using boron film reinforcement is that the laminate selected (±45°) is used almost exclusively for shear and not direct loads, in which direction it is particularly weak. Likewise, the combination of boron films plus 0° fibers would be superior to 0° fibers alone in the vicinity of a bolt hole. A comparable improvement in structural patterns in the practical range (0°/±45°/90°) through (0°/±45°/0°) would be really significant but the limited testing in this range has not been encouraging.

The real problem with load sharing in multiple-fastener patterns is not in the line-abreast (parallel load path) case tested in the paper, but in the line-astern (series load path) configuration with the bolts distributed along the load direction. Only an exceedingly poor insert would weaken the strength of a line of bolts perpendicular to the load, but any insert with even a slight further limit on the ultimate strain capacity along the load direction will inevitably effect a drastic reduction of joint strength. All the load tends then to be picked up by the outer bolts, leaving the inner ones very lightly loaded when failure occurs at the outer bolts. Consequently it would be premature to infer that boron film in fibrous composites strengthens all joints of multibolt configuration which are of practical interest to aerospace construction.

One further factor of importance in the use and testing of the B/PI film is the greatly reduced perimeter available for bonding, per unit cross-sectional area, in comparison with boron filaments. The load transfer problems at the ends of brittle filaments are known to be severe and have demanded a level of attention far greater than needed for conventional (ductile) metal structures. Otherwise, premature failures not reflecting the true capabilities of the materials have occurred. If the use of this new film is to lead to structural efficiencies exceeding those of conventional metal structure, even greater care will be needed in the design details. For example, significant strength increases would be demonstrated by eliminating the out-of-plane eccentricity in the load path shown in Fig. 4 of the paper.

Further testing of the more critical practical joint configurations will be necessary before the B/PI film can be assessed for bolted joint strength improvement. That such tests are in progress is indicated in Padawer's conclusion. The results will be awaited with interest throughout the composites industry. One advantage of the B/PI film in reinforcing single bolt holes is that it occupies less space than an equivalent integral build up of the basic filamentary composite laminate. This feature is of importance when maintaining loft surfaces without introducing excessive eccentricities in load paths.

Reference

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Comment on "A Compressibility Correction for Internal Flow Solutions"

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Nomenclature

Q = mass flow
q = mass flow ratio (defined in text)
R = gas constant
r = stagnation density ratio (defined in text)
T = temperature
V = velocity
γ = ratio of specific heats
ρ = density

Subscript

c = compressible
i = incompressible
t = stagnation
o = inlet or outlet plane

Introduction

POTENTIAL flow solutions for the blade-to-blade velocity distribution in turbomachines are now an accepted part of current design procedure. Various methods are available, and some of these generate incompressible solutions quite rapidly. Compressible solutions, however, involve considerably greater running time on computers. Hence, a simple form of compressibility correction suitable for turbomachine blade passages is an attractive prospect. Lieblein and Stockman[1] propose such a correction, of an essentially empirical nature. The present Note offers three examples of the use of their method, to demonstrate the degree to which it can be generally applied.

Analysis

It should be stated at the outset that a correction of this nature can only produce a compressible version corresponding to the same outlet gas angle as that of the original incompressible solution. This limitation can be a serious one, as experience has shown that for potential flow solutions to be correct, as judged by the usual criteria,* the outlet angles for incompressible and compressible cases must often be different.

The correction of Ref. 1 for blade surface velocities is given in the following form

$$V_c/V_i = (\rho_i/\bar{\rho}_c)^{V_i/\bar{V}_i} \qquad (1)$$

where subscripts i and c relate to incompressible and compressible values respectively, and the bar refers to average flow conditions across the passage at the same axial station. For the application of Eq. (1) its authors assume that stagnation density (ρ_i) and mass flow are the same for both incompressible and compressible cases. Those restrictions are, however, unnecessary and it is possible to use this correction in more general circumstances. Consider two solutions

a) Incompressible

mass flow = Q_i; density = ρ_i; at any axial station mean passage velocity = $\bar{V}_i$ and surface velocity = V_i

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* Either equal heights to the velocity peaks on suction and pressure surfaces immediately ahead of the trailing edge stagnation point, or, ignoring those peaks, extrapolations of the surface velocity curves which give zero trailing edge loading.

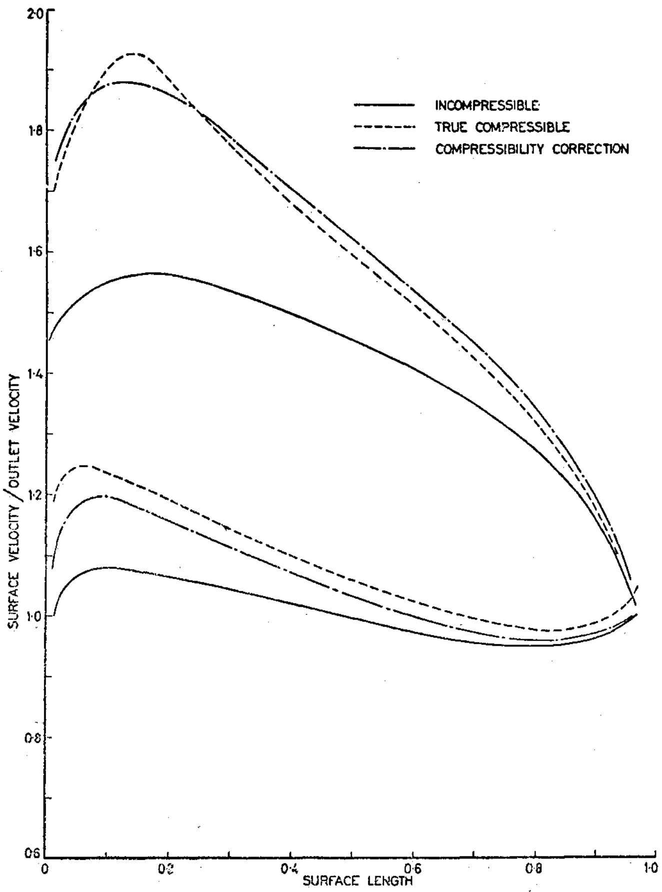

Fig. 1 Results for example A.

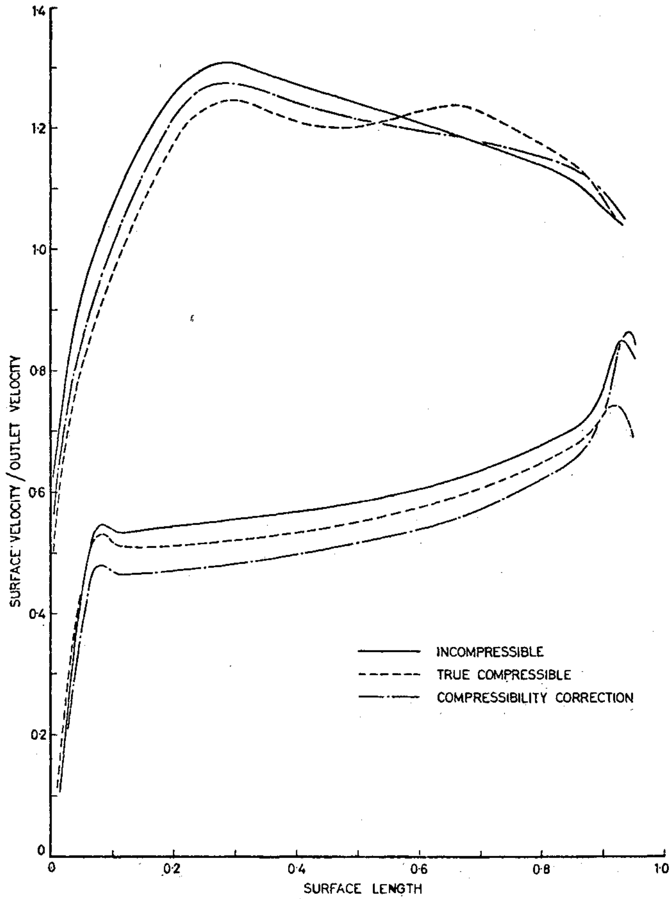

Fig. 2 Results for example B.

b) Compressible

mass flow = Q_c; stagnation density = $\rho_{c,t}$; at same axial station mean passage velocity = $\bar{V}_c$, surface velocity = V_c and mean density = $\bar{\rho}_c$

From continuity at any station, assuming equal flow angles in both incompressible and compressible cases,

$$Q_c/Q_i = \bar{\rho}_c \bar{V}_c/\rho_i \bar{V}_i \tag{2}$$

Writing mass flow ratio $Q_c/Q_i = q$ and stagnation density ratio $\rho_{c,t}/\rho_i = r$ Eq. (2) becomes

$$\bar{V}_c/\bar{V}_i = (q/r)\,\rho_{c,t}/\bar{\rho}_c \tag{2a}$$

Now q is of course independent of station and may be determined as follows: i) select the inlet or outlet blade Mach number (M_o) required for the compressible solution, ii) obtain the velocity ($V_{c,o}$) and density ratio ($\rho_{c,t}/\rho_{c,o}$) corresponding to M_o for given values of $T_{c,t}$ and γ, and iii) evaluate q from

$$q = r\left(\rho_{c,o}/\rho_{c,t}\right)\left(V_{c,o}/V_{i,o}\right) \tag{3}$$

Next, examine Eq. (1) in the light of the foregoing general relations between incompressible and compressible cases.

Table 1 Three compressible solutions

Example	Inlet gas angle	Outlet gas angle	Inlet Mach No.	Outlet Mach No.	Peak surface Mach No.
A	41.7	− 0.6	0.75		0.94
B	53.4	−63		0.67	0.85
C	0	−65		0.88	1

The authors of Ref. 1 state that "In Eq. (1) the density ratio term represents the effect of average Mach number," so it is clear that this should be written

$$V_c/V_i \propto (\rho_{c,t}/\bar{\rho}_c)^{V_i/\bar{V}_i} \tag{1a}$$

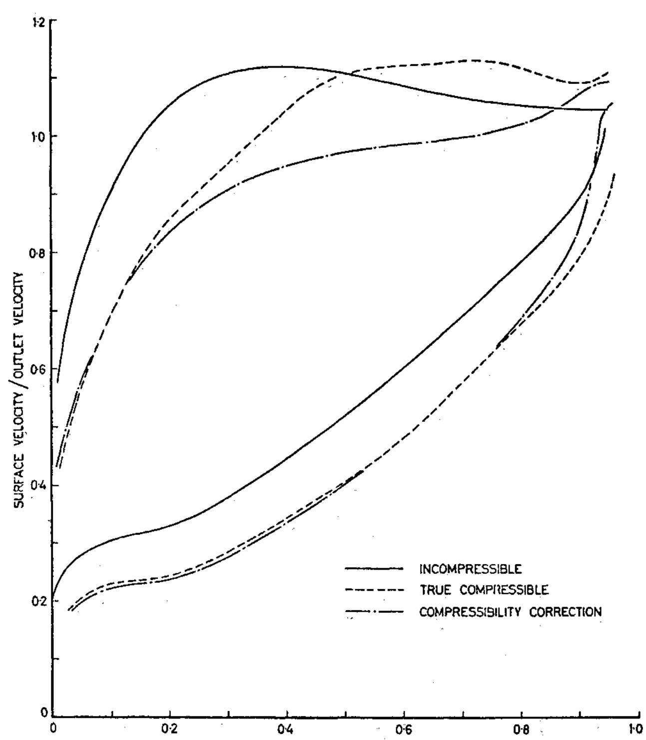

Fig. 3 Results for example C.

Velocity distributions are usually presented in the form of a ratio, viz: surface velocity/some reference velocity, that reference being commonly either inlet or outlet velocity. Therefore, given an incompressible velocity ratio, what is wanted is to predict the compressible velocity ratio at some particular flow conditions. Denoting the reference plane by suffix o, where the stream is assumed to be uniform, Eq. (1a) gives

$$V_{c,o}/V_{i,o} \propto \rho_{c,t}/\rho_{c,o}$$

But we know from Eq. (3) that

$$V_{c,o}/V_{i,o} = (q/r)\,\rho_{c,t}/\rho_{c,o}$$

Hence the constant of proportionality in Eq. (1a) is q/r, i.e.,

$$V_c/V_i = (q/r)(\rho_{c,t}/\bar{\rho}_c)^{V_i/\bar{V}_i} \tag{1b}$$

Expressed as velocity ratios the working form is then

$$\left(\frac{V}{V_o}\right)_c = \left(\frac{V}{V_o}\right)_i \frac{(\rho_{c,t}/\bar{\rho}_c)^{V_i/\bar{V}_i}}{\rho_{c,t}/\rho_{c,o}} \tag{1c}$$

Considering mean passage conditions at any station, the isentropic relations give for the compressible situation

$$\bar{V}_c^{\,2} = gRT_{c,t}\cdot\frac{2\gamma}{\gamma-1}\left[1-\left(\frac{\bar{\rho}_c}{\rho_{c,t}}\right)^{\gamma-1}\right] \tag{4}$$

Equations (2a) and (4) then produce

$$\left(\frac{\bar{\rho}_c}{\rho_{c,t}}\right)^2\left[1-\left(\frac{\bar{\rho}_c}{\rho_{c,t}}\right)^{\gamma-1}\right] = \frac{\gamma-1}{2\gamma}\,\frac{q^2\,\bar{V}_i^{\,2}}{r^2 gRT_{c,t}} \tag{5}$$

$\bar{V}_i$ is given by the incompressible solution, and hence the only unknown in Eq. (5) is $\bar{\rho}_c/\rho_{c,t}$, which can be evaluated at any axial station. It is then possible to apply Eq. (1c).

Thus use may be made of an incompressible solution with any value whatsoever of mass flow and density in order to predict the compressible solution for a particular mass flow and density.

When predicting a compressible solution in this way, a difficulty may arise in finding the correct outlet gas angle for that compressible condition. If this angle can be estimated, for example by some deviation rule, then the correction can be applied to an incompressible solution for that required angle.

Examples

The three examples presented relate to A) a compressor stator, at zero incidence, B) the rotor of a very low reaction turbine stage, at zero incidence, and C) a first stage turbine stator. Salient features of the design are given in Table 1. Potential flow solutions were obtained by Smith's matrix blade-to-blade computer program.[2,3]

Figures 1–3 show the results. It may be noted that the correction gives a blade loading which is different from that of the true compressible solution; it is too great in examples A and B, and too small in C. Since the correction is empirical in origin, this change is not surprising.

Example A is similar to that appearing in Ref. 1, although relating to a compressor rather than a turbine, in that the differences in velocity between incompressible and true compressible solutions are larger on the suction surface than on the pressure, and are uniform in sense throughout the blade chord. Consequently the good prediction shown for example A is further justification for the method when applied to such a situation.

In example B the velocity differences are much smaller and in general more nearly equal on suction and pressure surfaces. The prediction is certainly acceptable, but because of the relatively small effect of compressibility this is hardly a fair trial of the method.

For example C the true compressible solution shows a large chordwise shift of the suction peak, such that a significant proportion of loading is transferred towards the trailing edge. On the pressure surface the prediction is very good, but in these circumstances it is poor on the suction surface (where of course the major interest lies).

Conclusion

For the sort of velocity distributions from which the compressibility correction was derived, as illustrated in Ref. 1, it apparently works as well for a compressor as for a turbine. But it does not work satisfactorily when compressibility causes a major chordwise displacement of the suction peak. In order to estimate whether the correction will be satisfactory, it is thus necessary to have in advance some idea at least of what the true compressible solution looks like.

References

[1]Lieblein, S. and Stockman, N. O., "Compressibility Correction for Internal Flow Solutions," *Journal of Aircraft,* Vol. 9, No. 4, April 1972, p. 312.

[2]Smith, D. J. L. and Frost, D. H., "Calculation of the Flow past Turbomachine Blades," Paper 27, Institution of Mechanical Engineers. Thermodynamics and Fluid Mechanics Convention, Glasgow, Scotland, 1970.

[3]Smith, D. J. L., "Computer Solutions of Wu's Equations for the Compressible Flow through Turbomachines," International Symposium on Fluid Mechanics and Design of Turbomachinery, Pennsylvania State Univ., University Park, Pa., 1970.